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A STUDY ON APPLICATIONS OF FRACTIONAL CALCULUS OF MATHEMATICS

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ABSTRACT

The Chebyshev and Grüss type inequalities employing Appell hyper geometric functions are examples of fractional integral inequality. The Appell hyper geometric function and the extended Chebyshev functional are used to verify several novel fractional integral inequalities in this study. All the known fractional integral operators, Fractional integral operators, such as Saigo, Erdélyi-Kober, and Riemann–Liouville-type operators, are simply reduced to our fractional integral operator. Many scientific fields have relied heavily on the Mittag-Leffler function, such as biology and engineering as well as applied sciences. Using fractional order differential equations and fractional order integral equations, the Mittag-Leffler function may be constructed Mellin and Whittaker transforms of these functions in terms of the extended Wright hyper geometric function and Laplace transform were produced by extending Mittag-function Leffler's employing a generalised beta function. The Appell hyper-geometric function and the Lauricella function may be computed using extended Beta and Gamma functions.

Keywords: Applications, Fractional Calculus, Mathematics, and geometric functions.

INTRODUCTION

Fractional calculus has been a hot topic of study for a few years now, and it's just getting better. In many ways, fractional calculus and classical calculus were conceived at the same time. Fractional order derivative was initially proposed as a subject word in the seventeenth century. Even though the notation implies "calculus of fractions," it is noteworthy to note that today's fractional calculus subjects are far from being "calculus of fractions." Fractional calculus nowadays is best understood as "integration and differentiation to an arbitrary order," rather than "integration and differentiation." As a result of this question, fractional calculus was born. Integer factorials may be extended to complex number factorials, and vice versa, which is a well-known example of extending the meaning of an integer to its factorial complex equivalent. It's an issue of expanding meaning in generalized integration and differentiation. “Can the meaning of derivative of integral order d^n/dx^n be extended to have meaning, where nis any number - irrational, rational or complex”?

Integration and differentiation can be used to an infinite number of arbitrary order integrals and derivatives, as long as n is the operator's integral order d^n/dx^n to an arbitrary order. Calculus is commonly referred to as fractional calculus in the early stages of the expanded meaning since n was assumed to be a fraction. When it comes to the notion of a derivative, Leibnitz was the first person who tried to broaden it d^n/dx^n in the nth degree L' Hospital's question, "What if n be 1/ 2?" may have been a sign of naivete. Asked in 1695 if it would lead to paradox, Leibnitz replied: "It will lead to a paradox."

In the future, "valuable consequences" will be taken from this seeming paradox. In an essay published in 1819, Lacroix made the first reference of a derivative of arbitrary order. A article on fractional derivatives by Lacroix was published in 1819, making him the first mathematician to do so. Starting with $y = x^m$ where m is a positive integer, Lacroix easily developed the nth derivative

$$\frac{d^n y}{dx^n} = \frac{m!}{(m-n)!} x^{m-n} \quad m \geq n .$$

His formula for the generalized factorial, the gamma function, yields the answer.

$$\frac{d^n y}{dx^n} = \frac{\Gamma(m+1)}{\Gamma(m-n+1)} x^{m-n} .$$

He gave the example for m= 1 and $n = q$, and obtained same result as

$$\frac{d^q y}{dx^q} = \frac{x^{1-q}}{\Gamma(2-q)}$$

However, Lacroix's technique gave no hints as to how a derivative of arbitrary order may be used. To solve an integral equation that appeared during the proof of tautochrone, Niels Abel first used a fractional operation in 1823. In 1832, Joseph Liouville published the first comprehensive research on fractional calculus, perhaps inspired by Laplace and Fourier's short observations or Abel's demonstration. Given how beautiful Abel's answer was, it's safe to say that Liouville was compelled to make the first significant attempt to provide the first logical definition of the fractional derivative starting from two separate points of view. A posthumous article written by Riemann in 1847 while he was a student provided the definition of what is now known as the Riemann-Liouville integral. When it comes to the evolution of mathematical concepts, fractional calculus is no different from any other. That made some mathematicians wary of the overall idea of fractional operators. The book of Oldham and Spanier was published 279 years after L'Hospital introduced the issue, making it the earliest publication wholly devoted to fractional calculus. Ross provides a detailed account of the era between 1695 and 1974 in his book "The Fractional Calculus Bibliography in Chronological Order commentary".

There are several authors that have contributed to this topic, Ross' historical observations and Sneddon's surveys provide additional in-depth explanations. and Samko's massive encyclopaedia, Kilbas. There are several applications for fractional calculus, which has made this area of mathematics a popular choice for researchers in various domains. One may learn a lot about fractional calculus from the works of Oldham and Spanier; Nishimoto; Miller; Ross; Kiryakova; Samko; Kilbas; Marichev; Hilfe; Podulbny; among many others. According to Samko et al., the Riemann-Liouville fractional integral operator was defined.

$$I_{0+}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^{\infty} (x-s)^{\alpha-1} f(s)\, ds$$

By samko et al., the fractional integral operator of the Weyl type is defined.

$$I_{0-}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_x^{\infty} (x-s)^{\alpha-1} f(s)\, ds$$

Using the Erdelyi-Kober generalization fractional integral operators such as Weyl's Riemann-Liouville fractional integral operator

$$I_{0,x}^{\alpha,\beta} f(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\gamma)} \int_0^x (x-t)^{\gamma-1} t^{\beta} f(t)dt, \quad \alpha > 0, \beta \in R$$

$$\left(K_{x,\infty}^{\alpha,\beta} f(t)\right)(x) = \frac{x^{\beta}}{\Gamma(\alpha)} \int_x^{\infty} (t-x)^{\alpha-1} t^{-\alpha-\beta} f(t)dt, \qquad \alpha > 0, \beta \in R$$

Then generalize Erdelyi-Kober fractional integral operator incorporating Gaussian hyper geometric function by Saigo, for instance For $x > 0, \alpha, \beta, \gamma \in C$ and $R(\alpha) > 0$, we have

$$\left(I_{0,x}^{\alpha,\beta,\gamma} f(t)\right)(x) = \frac{x^{-\alpha-\beta}}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} \, {}_2F_1\left(\alpha+\beta, -\gamma; \alpha; 1-\frac{t}{x}\right) f(t)dt$$

$$\left(J_{x,\infty}^{\mu,\nu,\eta} f(t)\right)(x) = \frac{1}{\Gamma(\alpha)} \int_x^{\infty} (t-x)^{\alpha-1} t^{-\alpha-\beta} \, {}_2F_1\left(\alpha+\beta, -\gamma; \alpha; 1-\frac{x}{t}\right) f(t)dt$$

Only two definitions exist for fractional derivatives.

(i) The fractional derivative of order $\alpha \in (n-1,n)$ of a function $f(x)$ given by Riemann-Liouville.

$$\frac{d^{\alpha}}{dx^{\alpha}} f(x) = D^{n} I^{n-\alpha} f(x) \ , \quad D \equiv \frac{d}{dx}$$

(ii) The fractional derivative of order $\alpha \in (n-1,n)$ of a function $f(x)$ given by Caputo.

$$D^{\alpha} f(x) = I^{n-\alpha} D^{n} f(x) \ , \quad D \equiv \frac{d}{dx}$$

However, it has been shown that the definition of fractional derivative given by is beneficial for pure math issues but is more appropriate for applied math problems since it takes into consideration the beginning circumstances, which have a physical interpretation. In definition, if we take $\alpha \to n$, the fractional derivative $D\alpha$ becomes an ordinary nth derivative of $f(x)$.

MITTAGE –LEFFLER FUNCTION

The Mittag-Leffler function is naturally produced by fractional order differential and fractional order integral equations. Because of the significant rise in its interest in fundamental sciences like as physics, chemistry, biology, and engineering during the last two decades. There are several applications of the Mittag-Leffler approach, including studies of kinetic equations, random walks, Levy flights, and complex systems as well as practical applications including fluid flow, electric networks, probability, and statistical distribution theory. Gosta Mittag-Leffler, a Swedish mathematician, invented the function in 1903.

$$E_{\alpha}(z) = \sum_{n=0}^{\infty} \frac{z^{n}}{\Gamma(\alpha n + 1)}, \qquad z \in C, \alpha \geq 0 \ldots$$

It is direct generalization of the exponential function for $\alpha = 1$and for $0 < \alpha < 1$, it interpolate between exponential and the hyper geometric function 1/(1-z).

The generalization of E α (z) was further studied by Wiman in 1905 in the form

$$E_{\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{z^{n}}{\Gamma(\alpha n + \beta)}, \qquad \alpha, \beta \in C, \mathrm{Re}(\alpha) > 0, \mathrm{Re}(\beta) > 0$$

In 1971 Prabhakar introduced the function $E_{\alpha,\beta}^{\gamma}(z)$ in the form

$$E_{\alpha,\beta}^{\gamma}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!},$$

$$\forall\ \alpha,\beta,\gamma\in C, \operatorname{Re}(\alpha)>0, \operatorname{Re}(\beta)>0, \operatorname{Re}(\gamma)>0.$$

Where (γ) n is the pochhamer symbol

Extending Mittag-function Leffler's was introduced by Shukla and Prajapati in 2007, $E_{\alpha,\beta}^{\gamma,q}(z)$
This is defined as follows:

$$E_{\alpha,\beta}^{\gamma,q}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!},$$

where $\alpha,\beta,\gamma\in C, \operatorname{Re}(\alpha)>0, \operatorname{Re}(\beta)>0, \operatorname{Re}(\gamma)>0, q\in(0,1)\cup N$

$E_{\alpha,\beta}^{\gamma,q}(z)$ Converge absolutely $\forall\ z<1,\ q=\operatorname{Re}(\alpha)+1$.

Tariq O. Salim first introduced the function in 2009, $E_{\alpha,\beta}^{\gamma,\delta}(z)$

For $\alpha,\beta,\gamma\in C, \operatorname{Re}(\alpha)>0, \operatorname{Re}(\beta)>0, \operatorname{Re}(\gamma)>0, \operatorname{Re}(\delta)>0$ as follows

$$E_{\alpha,\beta}^{\gamma,\delta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_n}{\Gamma(\alpha n+\beta)}\frac{z^n}{(\delta)_n},$$

By Salim in 2012, the Mittag-Leffler function was given a new generalization as

$$E_{\alpha,\beta,p}^{\gamma,\delta,q}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{qn}}{\Gamma(\alpha n+\beta)\ (\delta)_{pn}}z^n,$$

where $\alpha,\beta,\gamma,\delta\in C$, $min[\operatorname{Re}(\alpha)>0, \operatorname{Re}(\beta)>0, \operatorname{Re}(\gamma)>0, \operatorname{Re}(\delta)>0]$.

In 2013, Mumtaz Ahmad Khan and Shakeel Ahmed presented a novel generalization of the Mittage-Leffler function.

$$E^{\mu,\rho,\gamma,q}_{\alpha,\beta,\upsilon,\sigma,\delta,p}(z)=\sum_{n=0}^{\infty}\frac{(\mu)_{\rho n}(\gamma)_{qn}}{\Gamma(\alpha n+\beta)(\upsilon)_{\sigma n}(\delta)_{pn}}z^n,$$

where $\mu,\rho,\gamma,\alpha,\beta,\upsilon,\sigma,\delta\in C; p,q>0$ and $q\le \mathrm{Re}(\alpha)+p$

$min[\mathrm{Re}(\alpha),\mathrm{Re}(\beta),\mathrm{Re}(\upsilon),\mathrm{Re}(\rho),\mathrm{Re}(\sigma),\mathrm{Re}(\delta),\mathrm{Re}(\mu),\mathrm{Re}(\gamma)>0]$.

K-ANALOGUE OF SOME SPECIAL FUNCTIONS

k-Pochhammer symbol, k-Gamma, k-Beta, and the k-Zeta functions, which are generalisations of pochhammer symbol, gamma, and beta functions, were researched by Rafael Diaz and Eddy Pariguan back in 2007. In addition, they showed various generalisations of the traditional gamma function, beta function, and pochhammer symbol. The integral representation was proven by Diaz and Pariguan for the $\Gamma k(z)$ and $B_k(x,y)$ functions. They have proven a number of characteristics of the hyper geometric function. Chris Kokologiannaki presented new characteristics and inequalities for the k-Gamma function throughout the year 2010 $\Gamma k(z)$, *k*-Beta function $B_k(x,y)$ and *k*-Zeta function $\varsigma_k(z,s)$. New and easy integral representations of several k-confluent hyper geometric functions and k-hyper geometric functions were introduced by S. Mubeen and G.M. Habibullah in 2012. It was in 2014 when S. Mubeen and A. Rehman demonstrated the k-Analogue of Vandermonde's theorem, which includes the binomial theorem as the limiting case and certain limit formulas utilising k-Symbols. A new k-Fractional integral based on the k-Gamma function was created in 2012 by S. Mubeen and G.M. Habibullah. It is used instead of the integral for k 1, Riemann-fractional Liouville's integral. Luis Guillermo Romero introduced it in 2013 and studied the connection with the k-Riemann–Liouville fractional integral. When Ruben A. and Gustavo Abel Dorrego met, they became friends. Cerutti invented the fractional derivative of two parameters in 2013, it was called the K-version. The following are the definitions of the k-functions in the examples above:

K-Gamma function

ForRe(z) > 0 and $\Gamma k(z)$ defined as a integral form

$$\Gamma_k(z)=\int_0^{\infty}t^{z-1}e^{-\frac{t^k}{k}}dt$$

k-Beta function

$$B_k(x,y)=\frac{1}{k}\int_0^1 t^{\frac{x}{k}-1}(1-t)^{\frac{y}{k}-1}dt$$

$$B_k(x,y)=\frac{\Gamma_k(x)\Gamma_k(y)}{\Gamma_k(x+y)},\ \mathrm{Re}(x)>0,\mathrm{Re}(y)>0\,.$$

where $(z)_{n,k}=z(z+k)(z+2k)......(z+(n-1)k);\ z\in C,k>0,k,n\in N.$

and $(z)_{n,k}=\dfrac{\Gamma_k(z+nk)}{\Gamma_k(z)}$

K-generalized hyper geometric function

$$_{m+1}F_{m,k}\left(\begin{matrix}(a,k),\left(\frac{b}{m},k\right),\left(\frac{b+k}{m},k\right),...,\left(\frac{b+(m-1)k}{m},k\right)\\ \\ \left(\frac{c}{m},k\right),\left(\frac{c+k}{m},k\right),...,\left(\frac{c+(m-1)k}{m},k\right)\end{matrix};z\right)$$

$$=\sum_{n=0}^{\infty}\frac{(a)_{n,k}\left(\frac{b}{m}\right)_{n,k}\left(\frac{b+k}{m}\right)_{n,k}\cdots\left(\frac{b+(m-1)k}{m}\right)_{n,k}}{\left(\frac{c}{m}\right)_{n,k}\left(\frac{c+k}{m}\right)_{n,k}\cdots\left(\frac{c+(m-1)k}{m}\right)_{n,k}}\frac{z^n}{n!}$$

$$=\sum_{n=0}^{\infty}\frac{(a)_{n,k}(b)_{mn,k}}{(c)_{mn,k}}\frac{z^n}{n!},$$

where $\mathrm{Re}(a)>0,\mathrm{Re}(c)>0,\mathrm{Re}(b)>0,k\ge 0,m\ge 1,m\in Z^+$

K-Gauss hyper geometric function

$$_2F_{1,k}(a,b;c;z)=\sum_{n=0}^{\infty}\frac{(a)_{n,k}(b)_{n,k}}{(c)_{n,k}}\frac{z^n}{n!}$$
$$={}_2F_1\left[(a,k),(b,k);(c,k);z\right]$$

for $k>0,|z|<1,\mathrm{Re}(c)>0,\mathrm{Re}(b)>0,\mathrm{Re}(a)>0,c\ne 0,-1,-2..$

For the conventional Mittag-Leffler function, Dorrego and Cerutti devised a k-Generalization in 2012.

$$E^{\gamma}_{k,\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{n,k}}{\Gamma_k(\alpha n+\beta)}\frac{z^n}{n!},$$

where $k>0,\alpha,\beta,\gamma\in C,\mathrm{Re}(\alpha)>0,\mathrm{Re}(\beta)>0.$

And $(\gamma)_{n,k}$ is *k*-Pochhamer symbol, If $k\to 1$the equation reduce to the classical Mittag-Leffler type function, and $\gamma = 1$, $k \to 1$ in the same equation we get two parameter Mittag-Leffler function that is Wright function , $W_{\alpha,\beta}$ defined as

$$W_{\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{1}{\Gamma(\alpha n+\beta)}\frac{z^n}{n!},$$

where $\alpha,\beta\in C,\mathrm{Re}(\alpha)>0,\mathrm{Re}(\beta)>0$.

The *k*-Wright function is defined as

$$W^{\gamma}_{k,\alpha,\beta}(z)=\sum_{n=0}^{\infty}\frac{(\gamma)_{n,k}}{\Gamma_k(\alpha n+\beta)}\frac{z^n}{(n!)^2},$$

where $k>0,\alpha,\beta,\gamma\in C,\mathrm{Re}(\alpha)>0,\mathrm{Re}(\beta)>0.$

There is an extension of the Riemann-Liouville fractional integral and k-Riemann-Liouville fractional derivative to k-Fractional integration, which was introduced by Mubeen and Habibullah in 2012.

Many scholars, including Mansour, Kokologiannaki, and Krasniqi, have proven a variety of conclusions about k-functions.

LITERATURE REVIEW

Diaz and Pariguan (2018)[1] introduced the k-gamma function Γ_k which is a one parameter deformation of the classical gamma function such that $\Gamma_k \to \Gamma$ as $k \to 1$ to symbolise the recurrence of certain phrases $x(x+k)(x+2k)\dots(x+(n-1)k)$.

Cerutti et al., (2017)[2] derived the generalization of k-Mittag-Leffler function known as the p-k Mittag-Leffler function denoted by ${}_pE^{\gamma,q}_{k,\alpha,\beta}(z)$ and defined in the form

$$ {}_pE^{\gamma,q}_{k,\alpha,\beta}(z) = \sum_{n=0}^{\infty} \frac{{}_p(\gamma)_{nq,k}}{{}_p\Gamma_k(n\alpha+\beta)} \frac{z^n}{n!} $$

M. Caputo, (2016)[3] owing to its wide range of applications in a variety of scientific and technical fields, such as fluid flow (rheology), diffusion, oscillation, anomalous diffusion, reaction-diffusion, and turbulence, turbulence diffuse transportation, electrical networks, polymer physics, chemical physics, E, the split-free calculation has become more and more popular over the last four decades.

F. Mainardi and G. Pagnini, (2013)[4] proved for derivatives of arbitrary order that $[D^p D^q f(x)]_{x_0}^{x} = [D^{p+q} f(x)]_{x_0}^{x}$ By the notion of partial computations he also solved certain differential equations.

Agarwal and Chand (2012)[5] the product I-function and Srivastava's polynomial were given new finite integrals. In the same year, more conclusions were drawn from Agarwal on Saigo's fractional calculus.

Nair (2009)[6] the route of the new fractional integral operator is introduced. The traditional Riemann Liouville fractional integration operator is generalized and the route parameter can also be reduced to the integral transformation Laplace α → 1.

Kiryakova (2008)[7] Author of a review paper providing the generalized fractional computational operators with a short history and shows that in various analytic domains all known fractional integral and differential operators have fallen under the generalized fractional computing scheme.

Kilbas et al., (2006)[8] An integral and a derivation can be described as arbitrary if they have distinct integral operators, which is what fractional calculus is all about. Arbitrary order calculus is another name for generalized comprehensive and differential calculus. Researchers like Fourier and Euler were interested in the fractional calculus as a topic of study. Mathematicians classify it as applicable. In the last 30 years scientists in physics, chemtics, quantitative bio-technology, engineering, image processing and signal theory, rheology, diffusion and transit theory and soon have identified various supplication requests.

Yu. Luchko, (2000)[9] extends several Hardy and Little wood findings across a large range, changes Mellin and creates a unique solution theorem.

$$g(x) = \int_a^x (x-t)^{\alpha-1} f(t)\, dt$$

F. Mainardi and R. Gorenflo (2000)[10] was the first mathematician to mention a derivative of arbitrary order in 1819. Thus for y = a, a ∈ R+, he showed that

$$\frac{d^{\frac{1}{2}}y}{dx^{\frac{1}{2}}} = \frac{\Gamma(a+1)}{\Gamma\left(a+\frac{1}{2}\right)} x^{a-\frac{1}{2}}$$

H.T. Davis, (1999)[11] Employed Volterra integral equations class fractional operators and compared many ratings to define fractional operators. He also proposed the following notation for the division operator

$${}_cD_x^{-v} f(x) = \frac{1}{\Gamma(v)} \int_c^x (x-t)^{v-1} f(t)\, dt$$

K.S. Miller and B. Ross (1993)[12] Fractional calculus permits any positive real order to be integrated and derived. It may be viewed as a field of mathematical analysis dealing with integral differential operators and equations in which the integrals of the turnover type and power law type kernels are displayed. It is strongly connected to the theory and special functions of pseudo different operators, including new classes of functions associated with fractional calculus, integral transformation, non-Gaussian processes, non-stochastic control theory, etc.

E.M. Wright, (1933)[13] tried to solve the fractional integral equation of brachistochrone, namely

$$\frac{1}{\Gamma\alpha} \int_0^\infty \frac{g(u)}{(x-u)^{1-\alpha}} du = f(x), \quad 0 < \alpha < 1$$

And arrived at the solution,

$$g(x) = \frac{1}{\Gamma(1-\alpha)} \frac{d}{dx} \left(\int_0^x \frac{f(u)}{(x-u)^{\alpha}} du \right)$$

OBJECTIVES OF THE STUDY

- To examine the Functional calculus and their Applications
- To investigate the Hyper geometric Function involved two or more Variables
- To analyze the Results of k-Hyper geometric Functions and k-Generalized Fractional Derivatives

RESEARCH METHODOLOGY

The current research is based on the introduction of a section of special features and the fractional calculus, while the remainder of the section extends and applies different generalized special features. fractional integral inequalities such as Shebyshev in the case of the synchronous function, asynchronous function, and the type of Grüss in which the hyper-geometric function is Appell. Here, we set up some new fractional integral inequalities, involving the hyper geometric function, which in the case of synchronous functions and asynchronous functions consider the extended function Chebyshev. Saigo and Maeda established a new category of fractional integral inequalities, related with the hyper geometrical Appell function, as a generalized fractional integral.

We constructed inequalities of the Grüss type that include generalized hyper geometric functions and have various characteristics. the functioning and generalizations of Mittag-Leffler. Due of its direct engagement in physics, biology, engineering, and applied science, this function has taken on a significant relevance in the past decades and a half. MittagLeffler function naturally happens as the solution of differential fractional order equations and integrated fractional order equations. Certain results are investigated with an extended beta feature and extended gamma feature on hyper-geometric function and Lauricella function. We also utilize Mridula Garg's generic lauricella function. Here we introduce several integrated theorems about the Lauricella function and describe the new widespread Lauricella function which includes Riemann-Liouville fractional integrated operators and partial differential operators, k-Hyper geometric characteristics, which comprise integration and differentiation, defined the generalized k-Fractional derivative as well, and produced several known results as particular instances. The universal k-fractional derivative was developed and interesting findings were achieved which included the image of the power function, the transforming Laplace and k-RiemannLiouville fractional integrated fractional with k-fractional derivatives.

CONCLUSION

Applications of the fractional calculus may be found in a wide range of technical and scientific domains such as electromagnetic and viscoelasticity as well as fluid mechanics and electrochemistry. Modeling physical and engineering processes using fractional differential equations has been proven to be the most effective method. The fractional derivative models are used to accurately model damping in systems that require it. Recently, a variety of analytical and numerical approaches have been suggested in these disciplines, as well as their application to brand-new issues.

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